# Subatomic Physics 2006/2007

Exam Subatomic Physics 2007/2008
Monday, 8 - February - 2008, 14:00 - 17:00

70 points
*( Please mark name on student number on every sheet.)*

## 1 Exam

### 1.1 Radioactive Isotopes (15 points)

The TRI$\mu$P facility at KVI in Groningen aims to study precisely nuclear $\beta$-decay. One of the particular interesting isotopes is ${}^{21}_{11}$Na. The isotope has a half-lifetime of $T_{1/2}$=23 sec.

(a) We know that there is only one stable Na isotope, which is $^{23}$Na. What kind of $\beta$-dacay is possible for $^{21}$Na.

(b) Give the equation describing this decay.

(c) The production of the isotope happens by shooting a ${}^{20}_{10}$Ne beam at energy $E_b$=23 MeV/u (energy per nucleon of the beam particles) on a deuterium ( ${}^{2}_{1}$H) gas target. Give the equation for this production reaction.

(d) The target is 10 cm long and it is kept at liquid nitrogen temperature (T=-77.5 K). The dimolecular deuterium molecule can be assumed to be an ideal gas which at standard conditions (temperature $T_0$= 273 K and pressure $p_0$= 1 bar) has a density of $6.023 \cdot 10^{23}$ particles per 22.4 dm$^3$. What is the cross section for the production reaction, if one produces $^{21}$Na at a rate of $\Gamma_0 = 3 \cdot 10^8$ particles per second with a $^{20}$Ne$^{+6}$ beam of 1 kW power (corresponding to $\Gamma_1 = 1.4 \cdot 10^{13}$ particles per second) ?

(e) Can you predict the (approximate) energy available for $\beta$-decay ?

(f) How does the energy spectrum of a $\beta$-decay look like (please give a qualitative sketch). Is there a difference for $\beta^+$- and $\beta^-$- decay? Explain the shape.

### 1.2 Neutrino experiments (10 points)

The nature of the neutrinos is an urgent issue in modern physics. Big detectors are operated or are being set up to detect neutrinos. Among them are the ICE-CUBE in Anartica and the KM3 in the Mediterranian Sea. Both use either 1 km$^3$ of antarctic ICE respectively sea water as active detector medium. For this, photomultipliers are deployed in the ice, respectively the Mediterranian water. Those detect Cherenkov light from reaction products of the neutrinos.

(a) Give an example of a reaction that could be used to detect a muon neutrino ($\nu_\mu$) respectively antineutrino($\overline{\nu_\mu}$). What interaction is needed for a (anti-)neutrino reaction?

(b) Can these detectors be used to detect neutrinos from tritium decay (maximum $\beta$-decay energie $\approx$ 17 keV?) Give one sentence of reasoning.

(c) At the highest up to date observed energies (about $10^{20}$eV) the cross section for neutrino reactions is estimated to be of order $10^{-31}$ cm$^2$ to $10^{-34}$cm$^2$. How big must the flux of neutrinos at these energies be at minimum to observe at least one event per day in the ICE-CUBE or KM3 detectors?

### 1.3 Weak Interactions (10 points)

The weak eigenstates and the mass/flavour eigenstates of quarks are not identical.

(a) How are they related to each other? Explain briefly.

(b) What is the situation for leptons? Explain briefly.

(c) What would change, if somebody would observe in future the process $\mu^+ \rightarrow e^+ + \gamma$.

(d) Give one example each of a (i) leptonic, (ii) semileptonic and (iii) non-leptonic weak interaction process. Draw the corresponding Feynman diagrams and explain how the coupling constants differ, which describe the vertices.

leptonic : involving only leptons & $W^\pm$, $Z^0$ boson ($e^- \rightarrow W^- + \nu_e$) $\mu^- + e^+ \rightarrow \nu_\mu + \bar{\nu}_e$
semileptonic : involving both leptons and hadrons $n \rightarrow p + e^- + \bar{\nu}_e$
non-leptonic : involving no leptons $\Lambda \rightarrow p + \pi^-$

### 1.4 Wave equations (10 points)

The Schroedinger equation is not relativistically co-variant.

(a) Which two relevant equations exist to improve on this problem?

(b) Which particles are described by each of the equations?

(c) The basic wave equations are first principles and cannot be derived from anything more fundamental, yet. Nevertheless, one can motivate them just like one can motivate the Schroedinger equation. Can you give the line of reasoning for this?

### 1.5 Allowed and Forbidden Processes ( 10 points)

Which of the following processes are allowed and which are forbidden. Please give reasons for your judgment. For allowed processes give the interaction.

(a) $e^+e^- \rightarrow \pi^+ + \pi^- + \pi^0 + pi^- + pi^+$

(b) $\overline{\nu_\mu} + p \rightarrow \mu^+ + n$

(c) $\nu_e + p \rightarrow e^+ + pi^0 + \Lambda^0$

(d) $e^- + e^- \rightarrow \mu^- + /mu^-$

(e) $^{76}Ge \rightarrow ^{76}Se + e^- + e^-$ neutrinoless double $\beta$-decay!

(f) $p \rightarrow \pi^+ + e^- + e^+ + \gamma$

(g) $\mu^- \rightarrow e^- + e^+ + e^-$

(h) $J/\Psi(2^1S_0) \rightarrow J/\Psi(1^1S_0) + 3\gamma$ (remember: $J/\Psi = (c\bar{c})$ bound state)

(i) $p + \overline{p} \rightarrow b\overline{b}$

(h) $p +^8 Be \rightarrow^8 Be + n + \pi^+ + \pi^- + \pi^+$

### 1.6 Radioactivity (15 points)

One Curie (Ci) used to be the unit for (radio)-activity. It was defined as the $\alpha$-decay activity (for a certain $\alpha$ energy) of 1 g of the very long living $^{226}_{88}Ra$ isotope (1 Ci $= 3.7 \cdot 10^{10}$ decays per second).

(a) What is the half-lifetime ($T_{Ra}$) of $^{226}Ra$?

(b) $^{226}$Ra decays into $^{222}$Rn. Give the reaction equation and estimate the $\alpha$- energy.

(c) Please sketch the $\alpha$ energy spectrum.

(d) $^{222}$Rn lives 3.8 days ($T_{Rn} = 3.3 \cdot 10^5$ seconds). Staring from a clean sample of only $^{226}$Ra, how much $^{222}$Rn is available in a day, a month and a year?

# 2 Constants, Relations and Conversion Factors

### 2.0.1 Constants that could be of Relevance

| | | | |
|---|---|---|---|
| Speed of light | $c$ | $2.998 \cdot 10^8$ | m/s |
| Planck constant | $h$ | $4.136 \cdot 10^{-24}$ | GeVs |
| | $\hbar = \frac{h}{2\cdot\pi}$ | $6.582 \cdot 10^{-25}$ | GeV/c |
| Electron charge | $e$ | $1.602 \cdot 10{-}19$ | C |
| Electron mass | $m_e$ | 0.510998918(44) | MeV/c$^2$ |
| Proton mass | $m_p$ | 938.272029(80) | MeV/c$^2$ |
| Neutron mass | $m_n$ | 939.565360(81) | MeV/c$^2$ |
| Deuteron mass | $m_d$ | 1875.61282(16) | MeV/c$^2$ |
| Alpha particle mass | $m_\alpha$ | 3727.37917(32) | MeV/c$^2$ |
| Electron neutrino mass | $m_{\nu_e}$ | $< 2.2$ | eV/c$^2$ |
| Muon mass | $m_\mu$ | 105.658369(9) | MeV/c$^2$ |
| Charged Pion mass | $m_{\pi^\pm}$ | 139.57018(35) | MeV/c$^2$ |
| Avogadro's number | $N_A$ | $6.02214179(30) \cdot 10^{23}$ | mol$^{-1}$ |

### 2.0.2 Relations

**Mass of Atom (Bethe-Weizaecker):**

$M(A,Z) = Nm_n + Zm_p + Zm_e - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}$

with

$a_v = 15.67 MeV/c^2$

$a_s = 17.23 MeV/c^2$

$a_c = 0.714 MeV/c^2$

$a_a = 93.15 MeV/c^2$

$\delta = 0$ (odd A) or -11.2 MeV/c$^2$ (Z and N even) or +11.2 MeV/c$^2$ (Z and N odd)

**Schroedinger equation:**

$$\frac{-\hbar}{i}\frac{\partial}{\partial t}\Psi(\vec{r},t) = [-\hbar^2/2m\vec{\nabla}^2 + V(\vec{r},t)]\Psi(\vec{r},t) \tag{1}$$

**classical momentum-energy relation free particle**

$$E = \frac{p^2}{2m} \tag{2}$$

**classical momentum-energy relation free particle**

$$E^2 = p^2c^2 + m_0^2c^4 \tag{3}$$

### 2.0.3 Conversion Factors

| | | | |
|---|---|---|---|
| Electronvolt | $eV$ | $1.60217653(14) \cdot 10^{-19}$ | J |
| Tesla | $T$ | $0.561 \cdot 10^{30}$ | MeV/(c$^2\cdot$ C $\cdot$ s) |
| Kilogram | $kg$ | $5.60958896(48) \cdot 10^{35}$ | eV/c$^2$ |
| barn | $b$ | $1 \cdot 10^{-28}$ | m$^2$ |

*Note: For some of the questions different approaches are possible, such that you may not necessarily need all of the given constants and equations. Unless differently stated, the final results are sufficient, if if given to 2 significant figures (2 leading digits).*